PHYS 1131 TEST 1R.

1 (17)

(a) $\underset{\sim}{r} = r\cos\theta\, \underset{\sim}{i} + r\sin\theta\, \underset{\sim}{j}$

$= r\cos(wt)\,\underset{\sim}{i} + r\sin(wt)\,\underset{\sim}{j}$ (3).

(b) $\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = r(-w)\sin wt\, \underset{\sim}{i} + rw\cos wt\, \underset{\sim}{j}$

$= rw(-\sin wt\, \underset{\sim}{i} + \cos wt\, \underset{\sim}{j})$ (1).

$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = rw(-w\cos wt\, \underset{\sim}{i} - w\sin wt\, \underset{\sim}{j})$

$= -rw^2(\cos wt\, \underset{\sim}{i} + \sin wt\, \underset{\sim}{j})$ (1).

$= -w^2 \underset{\sim}{r}$ (1).

(c) $\underset{\sim}{v}\cdot\underset{\sim}{r} = r^2 w(-\cos wt \sin wt + \sin wt \cos wt) = 0$ (2)

$\underset{\sim}{v}$ is perpendicular to $\underset{\sim}{r}$ (1).

(d) $r = 1.2\,m$
$y - y_0 = 1.8\,m$
$x - x_0 = 9.0\,m$.

$y - y_0 = \frac{1}{2}at^2 \Rightarrow t = \sqrt{2(y-y_0)/a} = \sqrt{2\times 1.8/9.8} = 0.606\,s$.

$x - x_0 = v_0 t \Rightarrow v_0 = (x-x_0)/t = 9.0/0.606 = 14.9\,ms^{-1}$.

$\Rightarrow$ centripetal acceleration $a_c = v^2/r = (14.9)^2/1.2 = 180\,ms^{-2}$ (5)

(e) Project onto x-axis, for instance.
$\Rightarrow$ from part (b):

$a = \frac{d^2x}{dt^2} = -w^2 x$, $x = r\cos wt$

This is simple harmonic motion (3).

2. (17)

(a) $K = \frac{1}{2}mv^2$

Force that keeps satellite in circular orbit

$$F = -\frac{GmM_e}{r^2} = -m v^2/r$$

$$\Rightarrow K = \frac{1}{2}mv^2 = \frac{GmM_e}{2r}.$$ (3)

(b) $\frac{1}{2}mv_0^2 - \frac{GmM_e}{R_e} = \frac{1}{2}mv^2 - \frac{GmM_e}{r}$

$$= \frac{GmM_e}{2r} - \frac{GmM_e}{r}$$

$$= -\frac{GmM_e}{2r}.$$

$$\Rightarrow \frac{1}{2}\cancel{m}v_0^2 = G\cancel{m}M_e\left(\frac{1}{R_e} - \frac{1}{2r}\right)$$

$$\Rightarrow v_0 = \sqrt{2GM_e\left(\frac{1}{R_e} - \frac{1}{2r}\right)}$$ (4)

When $r \to \infty$, $v_0 \to \sqrt{2GM_e/R_e}$ escape speed from Earth. (1)

(c) $E = K_A + K_B + K_e + U_{Ae} + U_{Be} + U_{AB}$, A, B – satellite

$\simeq K_A + K_B + U_{Ae} + U_{Be}$

$= 2(K + U)$, $K_A = K_B = K$, $U_{Ae} = U_{Be} = U$

$= \cancel{2}\left(-\frac{GmM_e}{\cancel{2}r}\right)$

$= -\frac{GmM_e}{r}$ (3)

(d) Cons. of momentum $mv - mv = 0 = 2mV$.
$\Rightarrow$ velocity of wreckage immediately following collision $V = 0$

$\Rightarrow \quad E \simeq U_{Ae} + U_{Be} = 2U = -2\frac{GmM_e}{r}$ (4)

(e) Wreckage falls toward Earth. As r decreases, U becomes smaller, and so the kinetic energy K gets larger; wreckage accelerates. Since the angular momentum is zero, wreckage falls directly toward centre of Earth (no rotation). (5)

3. (16)

(a) $mgh = \frac{1}{2}mv_A^2$ energy cons.

$\Rightarrow v_A = \sqrt{2gh}$ (2)

(b) $mgh - F_f l = \frac{1}{2}mv_B^2$

$\Rightarrow mgh - mg\mu_k l = \frac{1}{2}mv_B^2$

$\Rightarrow v_B = \sqrt{2g(h-\mu_k l)}$ (3)

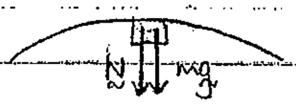
or can use work-energy theorem

$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = W = -mg\mu_k l$

(c) (i) $\frac{1}{2}mv_B^2 = \frac{1}{2}mv_C^2 + mg(2r)$

$\Rightarrow v_C = \sqrt{v_B^2 - 4gr}$

$= \sqrt{2g(h-\mu_k l - 2r)}$ (3)

(ii) 

(2)

(iii) Forces acting at point C: $\sum F = N + mg = m v_C^2/r$

Block maintains contact with track when $N > 0$

i.e. $N = mv_C^2/r - mg > 0$

$\Rightarrow mv_C^2/r > mg$

$\Rightarrow v_C^2 > rg$

(from part (i)) $\Rightarrow 2g(h-\mu_k l - 2r) > rg$

(iii) cont.

$$\Rightarrow \quad h - \mu_k l - 2r > r/2$$

$$\Rightarrow \quad h > 5r/2 + \mu_k l$$

$l = 3\,cm, \; r = 5\,cm$

$$\Rightarrow h > 5/2 \cdot 5 + 0.3 \cdot 3$$ (4)

$$= 13\ cm$$

(d) $\frac{1}{2} m v_D^2 = \frac{1}{2} k x^2$

$v_D = v_B \Rightarrow \quad m v_B^2 = k x^2$

$$\Rightarrow k = m v_B^2 / x^2 = m \cdot 2g(h - \mu_k l)/x^2$$

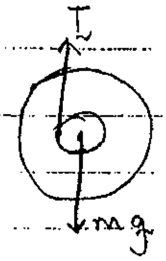
$$= 0.025 \times 2 \times 9.8 (0.18 - 0.3 \times 0.03)/(0.03)^2$$

$$= 93\ Nm^{-1}$$ (2)

17

4.

(a)



The tension in the string produces a torque about the centre of mass (3)

(b) $\sum \tau = R_0 T = I\alpha \quad (= I a / R_0)$ (3)

(c) $\sum F = mg - T = ma$ (3)

(d) (i) $mg - T = ma = m\, R_0^2\, T / I$

$$\Rightarrow T(1 + mR_0^2/I) = mg$$

$$\Rightarrow T = \frac{mg}{1+mR_0^2/I}$$

$$= \frac{mg}{1+\cancel{m}R_0^2/(\frac{1}{2}\cancel{m}R^2)}$$ (2)

$$= \frac{mg}{1+2R_0^2/R^2}$$

(ii) $a = (mg-T)/m = g - \frac{g}{1+2R_0^2/R^2}$

$$= \frac{g(\cancel{1}+2R_0^2/R^2 - \cancel{1})}{1+2R_0^2/R^2}$$

$$= \frac{g\;2R_0^2/R^2}{1+2R_0^2/R^2}$$ (2)

(e) Cons. energy

$$\tfrac{1}{2}I\omega_0^2 = mgl \Rightarrow \omega_0 = \sqrt{2mgl/I}$$

$$\Rightarrow \omega_0 = \sqrt{2\cancel{m}gl/\tfrac{1}{2}\cancel{m}R^2} = \sqrt{4gl/R^2} = \sqrt{4\times 9.8\times 0.8/(0.03)^2}$$ (4)

$= 187$ rad s^{-1}

5.

(a) Initial: $x_{cm} = l + L/2$

Final: $(M+50m)\,x_{cm} = (d+L/2)M + (d+L)(50m)$

Centre of mass x_{cm} is same in initial and final case (no ext. forces act)

$$\Rightarrow (M+50m)(l+L/2) = (d+L/2)M + (d+L)\,50m$$

$$= d(M+50m) + LM/2 + 50mL$$

$$\Rightarrow d = \frac{1}{M+50m}\left[(M+50m)(l+L/2) - LM/2 - 50mL\right]$$

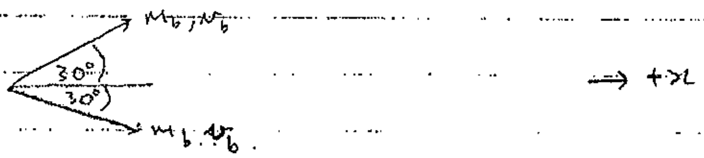
$$= \frac{1}{M+50m}\left[(M+50m)l - 25mL\right]$$

$$= \frac{1}{3500+50\times75}\left[(3500+50\times75)8 - 25\times75\times30\right]$$

$$= 24\text{ cm}$$ (6)

(b) speed of ferry relative to the water is zero. (since centre of mass was originally at rest and there are no ext. forces) (2)

(c)



Cons. of momentum

$$0 = 2m_b v_{bx} + (M+50m-2m_b)V$$

$$\Rightarrow V = -\frac{1}{M+50m-2m_b}\left(2m_b v_b \cos 30°\right) \text{ toward croc.}$$

$$= -\frac{1}{3500+50\times75-400}\left(2\times200\times20\cos30°\right)$$

$$= 0.18\text{ ms}^{-1} \text{ away from croc.}$$ (6)

(d) Final speed according to observer

$V' = \underline{V} + v_{current}$, $v_{current} = 3 km/h$

$= 3 \times 10^3 / (60 \times 60)$

$= 0.833 ms^{-1}$.

$\Rightarrow V' = 0.180 + 0.833$

$= 1.0 ms^{-1}$. (3)

16

6.

(a) $I = M/L \int_0^L x^2 \, dx = M/L \cdot L^3/3 = ML^2/3$ (2)

(b) $I_{prop} = 3 \cdot I = ML^2 = 220 \times 5^2 = 5500 \text{ kg m}^2$. (2).

(c) $I_{prop+block} = I_{prop} + I_{block} = ML^2 + ml^2 = 5500 + 25 \times 2^2$

$= 5600 \text{ kg m}^2$ (2).

(d) $\tau = I\alpha \Rightarrow \alpha = \tau / I = 7000/5600 = 1.25 \text{ rad s}^{-2}$.

$\omega - \omega_0 = \alpha t$

$\Rightarrow t = (\omega - \omega_0)/\alpha$, $\omega = 2000 \text{ rpm}$

$= 2000 \times 2\pi / 60$

$= 209.4 \text{ rad s}^{-1}$.

$\Rightarrow t = 209.4 / 1.25$

$= 168 \text{ s}$. (4)

(e). Block has travelled a distance

$s = l\theta$, $\theta = \cancel{\theta_0} + \cancel{\omega_0 t} + \frac{1}{2}\alpha t^2$.

$= \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 1.25 \times 168^2 = 17640 \text{ rad}$

$\Rightarrow s = 2 \times 17640 = 35 \text{ km}$. (3).

(f). $I_i \omega_i = I_f \omega_f$

$\Rightarrow \omega_f = I_i \omega_i / I_f = 5600 \times 209.4 / (5500 + 25 \times 2.5^2)$

$= 207 \text{ rad s}^{-1}$ $(= 1980 \text{ rpm})$

(3).